Problem Set 2

All numbered problems come from Stephen Boyd and Lieven Vandenberghe's Additional exercises for Convex Optimization. Use the version on the course website so problem numbers match. All data is posted on the course website.

Problems

1. Give an explicit solution of each of the following LPs

   a) Minimizing a linear function over a rectangle

   $$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & l \leq x \leq u. \end{array}$$

   b) Minimizing a linear function over the probability simplex

   $$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & \mathbf{1}^T x = 1, \quad x \geq 0. \end{array}$$

   c) Minimizing a linear function over a unit box with a total budget constraint.

   $$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & \mathbf{1}^T x = k, \quad 0 \leq x \leq 1, \end{array}$$

   where $k \in \{0, 1, \ldots, n\}$. What if we constrain $x \in \{0, 1\}^n$? What if we change the equality constraint to an inequality?

Choose at least four of the problems below. If you do all five correctly, you will get 0.5pts of extra credit on your homework score, which cannot exceed 12pts total across all three problem sets.

2. *20.14 Electric vehicle charging.* (p. 249)
3. *16.8 Well that was a bit roundabout.* (p. 181)
4. *21.22 Optimal policies for and shipments between two blood banks.* (p. 268)
5. *17.17 Portfolio optimization with qualitative return forecasts.* (p. 198)
   You don't have to refer to the textbook (but certainly can). The problem is

   $$\begin{array}{ll} \text{maximize} & R^{\text{wc}} \\ \text{subject to} & \mathbf{1}^T x = 1, \quad x^T \Sigma x \leq \sigma_{\text{max}}^2, \end{array}$$

   where R^{wc} is defined in the problem. Hint: you may want to split x into its positive and negative parts and/or use one of your answers from problem 1.
6. *7.18 Maximum likelihood estimation of a log-concave distribution.* (p. 99)